SCOPE OF THE QUALIFICATION EXAMINATION - MATHEMATICS AND COMPUTER SCIENCES

Doctoral School of Exact and Natural Sciences

MATHEMATICS

Structure of the exam: 8 problems in the discipline of Mathematics within the pool of 16 problems proposed together with the discipline of Computer Science. For the final result, 4 best evaluated problems are chosen from the 16 proposed problems. The following list of topics is indicative only; exam problems may partly relate to other topics within the same general thematic range.

1. Mathematical analysis – functions of one variable
Examples of topics: real and complex numbers and their properties, sequences and their limits, Bolzano-Weierstrass's theorem, Cauchy's condition, criteria of existence of a limit, series of real and complex numbers, convergence criteria for series, series conditionally and absolutely convergent, multiplication of series, continuity and uniform continuity of functions, properties of continuous functions defined on compact sets, Darboux's property, differential calculus of real functions of one variable, Rolle's I Lagrange's theorems, using derivatives and limit when graphing a function, series of functions, pointwise and uniform convergence, power series, radius and circle of convergence, Taylor's expansion, indefinite integral, Riemann integral, improper integral.

2. Mathematical analysis – functions of many variables
Examples of topics: partial derivatives and directional derivative, gradient, Jacobian, extrema of functions of many variables, implicit functions, Lagrange's multipliers, theory of Lebesgue measure and integral, interchange of integration with the limit, Fubini's theorem, curvilinear and surface integrals, differential manifolds and differential forms.

3. Analytic functions
Examples of topics: Holomorphic functions, Cauchy-Riemann equations, the winding number of a closed curve, Cauchy integral formula, identity principle, maximum modulus principle, Laurent series, Cauchy's residue theorem, argument principle, Rouché's and Hurwitz's theorems, Schwarz lemma.

4. Probability theory and statistics
Examples of topics: conditional probability, independence, random variables and their parameters, conditional expectation, Markov chains, types of convergence of sequences of random variables, laws of large numbers and the central limit theorem. Elements of statistics: estimators and their properties, testing hypotheses, linear regression.

5. Geometry and linear algebra
Examples of topics: determinants and linear equations, linear and affine spaces, linear transformations, eigenvalues and eigenvectors, Jordan's theorem, bilinear and quadratic forms, Sylvester's criterion, inner products, selfadjoint operators.

6. Algebra
Examples of topics: groups, cyclic groups, groups of permutations, group homomorphisms, kernel, normal subgroup and quotient group, Lagrange's theorem about the order of a subgroup, commutative rings, ideals, maximal and prime ideals, homomorphisms of rings, zero divisors, invertible elements, field of fractions, fields, prime field, characteristic of a field, algebraically closed field, fundamental theorem of algebra, roots of unity.

7. Topology
Examples of topics: metric and topological spaces, methods of defining a topology, Tikhonov's theorem, continuous mappings, Tietze's theorem, connected spaces, compact spaces, complete spaces, Cantor set and its properties, Baire's theorem, Banach's and Brouwer's fixed point theorems, fundamental group, compact surfaces.

8. Ordinary differential equations
Examples of topics: existence and uniqueness of solutions of ordinary differential equations, solving ordinary differential equations of one real variable, linear ordinary differential equations and systems of first order linear differential equations with constant coefficients and solving them, s, higher order linear equations, harmonic oscillator equation with and without friction, elements of qualitative theory of differential equations, logistic equation, Lotka-Volterra prey-predator model, various concepts of the stability of solutions.

9. Functional analysis
Examples of topics: Banach spaces, bounded linear functionals/operators, dual spaces to c0, lp, function spaces Lp and the space of continuous functions C[a,b], Hahn-Banach theorem, separation theorems, compact operators, Riesz-Schauder theorem, Hilbert spaces, spectral theorem for compact self-adjoint operators, Banach-Steinhaus theorem, closed graph theorem and open mapping theorem.

10. Computational mathematics
Numerical matrix decompositions in application to solving systems of linear algebraic equations, sensitivity of numerical solutions of systems of linear equations with respect to data disturbances, methods of interpolation and approximation of functions, numerical integration, numerical methods of solving nonlinear algebraic and differential equations.

COMPUTER SCIENCES

Structure of the exam: 8 problems in the discipline of Computer Science within the pool of 16 problems proposed together with the discipline of Mathematics. For the final result, 4 best evaluated problems are chosen from the 16 proposed problems. The following list of topics is indicative only; exam problems may partly relate to other topics within the same general thematic range.

1. Programming languages
Examples of topics: language constructs encountered in imperative, objectoriented, functional and logic programming languages, semantics of programming languages, software verification techniques, type systems.

2. Discrete mathematics
Examples of topics: combinatorics, elements of graph theory, elements of number theory, asymptotics.

3. Probability theory and statistics
Examples of topics: conditional probability, independence, random variables and their parameters, conditional expectation, Markov chains, types of convergence of sequences of random variables, laws of large numbers and the central limit theorem. Elements of statistics: estimators and their properties, testing hypotheses, linear regression.

4. Algorithms and data structures
Examples of topics: knowledge and ability to create algorithms with provable guarantees on pessimistic (or expected) running time and on correctness, dynamic programming, sorting and selection, basic data

structures (e.g. dictionary, priority queue), graph algorithms (e.g. minimal spanning tree, maximal matching, maximal flow) and text algorithms, linear programming.

5. Logic and databases
Examples of topics: Propositional logic, first- and second-order logic, relational algebra, SQL, intuitionism, expressivity and non-expressivity, decidability and complexity of logical theories.

6. Automata and formal languages
Examples of topics: Finite automata, regular expressions, context-free grammars, pushdown automata, recognizability and non-recognizability, closure properties, decidability and complexity of the problems of belonging to a language, nonemptiness, language inclusion.

7. Computation theory and computational complexity
Examples of topics: Turing machines, decidable and undecidable problems, complexity classes P, NP, PSPACE and others, hardness and completeness, Boolean circuits and complexity classes based on them, Las Vegas and Monte Carlo randomized algorithms, approximate algorithms.

8. Concurrent and distributed programming, computer systems
Examples of topics: Models of concurrency, communication and synchronization mechanisms, paradigms of distributed computation, data integrity models, proving correctness of concurrent programs, basic problems of concurrency and algorithms to solve them, computer system architecture, processes and mechanisms of process management, memory hierarchy and
data storage, process communication and network protocols, computer system security.

9. Bioinformatics
Examples of topics: Sequence alignments. models of sequence evolution, phylogenetic trees, clustering of molecular sequences, hidden Markov models, efficient data structures for matches with errors, de Bruijn graphs.