

Doctoral School, physical sciences, exam

Present the final results of the calculations with an accuracy of 3 or 2 significant digits, after appropriate rounding, for example $1.23456 \cdot 10^{-19} \approx 1.23 \cdot 10^{-19}$ or $1.2 \cdot 10^{-19}$.

Values of selected constants

speed of light in vacuum	$c \approx 3 \cdot 10^8$ m/s
elementary charge	$e \approx 1.6 \cdot 10^{-19}$ C
Coulomb constant	$k_e \approx 9 \cdot 10^9$ N m ² /C ²
Planck constant	$h \approx 6.63 \cdot 10^{-34}$ J s $\approx 4.14 \cdot 10^{-15}$ eV s
reduced Planck constant	$\hbar = \frac{h}{2\pi} \approx 1.05 \cdot 10^{-34}$ J s $\approx 6.58 \cdot 10^{-16}$ eV s
gravitational constant	$G \approx 6.67 \cdot 10^{-11}$ N m ² /kg ²
Avogadro constant	$N_A \approx 6 \cdot 10^{23}$ mol ⁻¹
gas constant	$R \approx 8.3$ J/(K mol)
Boltzmann constant	$k_B \approx 1.38 \cdot 10^{-23}$ J/K $\approx 8.62 \cdot 10^{-5}$ eV/K
Rydberg constant	$R_\infty \approx 1.1 \cdot 10^7$ m ⁻¹
rydberg	Ry ≈ 13.6 eV
electron mass	$m_e \approx 9.11 \cdot 10^{-31}$ kg ≈ 511 keV/c ²
proton mass	$m_p \approx 1.67 \cdot 10^{-27}$ kg ≈ 938 MeV/c ²

Problems 1–7 are easier.

Send solutions of all these problems!

For each of them you can get 4 points.

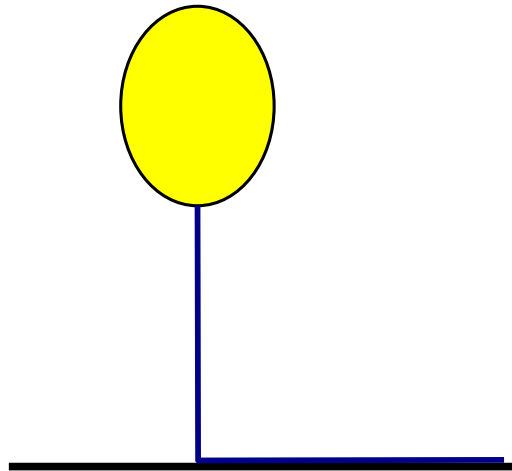
Problems 8–13 are more difficult.

Send solutions of only two of them!

For each of these solutions you can get 6 points.

1 Problem – A tethered balloon

An empty balloon is attached to the end of a thin, homogeneous, 10-meter long flexible string. The mass of the thin skin of the balloon is $m_p = 75\text{g}$. The linear density of the string is equal to $\lambda = 10\text{g/m}$. Then the balloon is filled with $n = 4$ moles of helium-4. The helium temperature is $T = 300\text{K}$ and the pressure inside the balloon is $p = 1100\text{hPa}$. After that, the balloon is raised, and subsequently released. The balloon falls a dozen centimeters and then stays motionless in the air. Part of the string that is attached to the balloon hangs vertically while the remaining part of the string lies horizontally, stretched along a straight line on the floor (as shown in the figure). The air density in which the balloon floats is equal to $\rho = 1.2\text{kg/m}^3$.



Calculate the height above the floor at which the balloon floats, i.e. the length h of the piece of string (connected to the balloon) that hangs vertically.

2 Problem – Hearing and motion

For sound frequencies in the range 500–3000 Hz, the human hearing senses a change in frequency if this change is greater than 0.1–0.35%. Assume that the speed of sound in the air of the laboratory is $u = 340\text{m/s}$, and the experimenter can perceive a change in sound frequency of $\delta = 0.3\%$. The experiment uses a source emitting sound waves with a given fixed frequency. In the following cases, the experimenter compares her or his hearing experience with the case when she or he and the sound source are at rest in the laboratory frame.

- Calculate the minimum speed of the sound source approaching the stationary experimenter for which the experimenter may hear a change in frequency.
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3 Problem – Process $\gamma e^- \rightarrow \gamma e^-$

A photon with a wavelength of λ_1 scatters on a free, resting electron. In the final state we observe a photon with a wavelength of λ_2 at an angle of α relative to the direction of the initial photon, and the electron.

Calculate the dependence of the difference $\lambda_2 - \lambda_1$ on the angle α .

4 Problem – Frame and magnetic field

From a 70 cm long thin wire with resistance 20Ω a closed circuit was made – the rectangular frame with sides 15 cm and 20 cm. The frame was lying on a horizontal table, in a vertical, uniform magnetic induction of 1.1 T. During a certain period of time, the magnetic induction value decreased to 0.3 T. Assume that the magnetic field was always vertical, uniform, downward, and the frame did not move. Ignore the magnetic field created by the flow of the current in the frame.

Calculate the absolute value of the charge that passed through the wire cross-section in the time interval when the magnetic induction value was decreasing.

5 Problem – Photoelectric effect

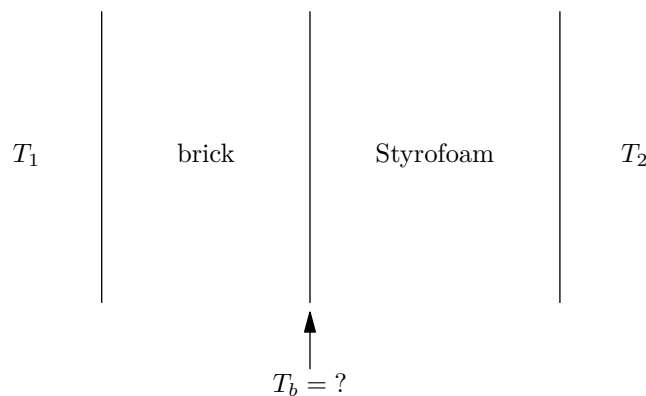
The metal plate was illuminated with electromagnetic radiation of 220 nm wavelength. The maximum kinetic energy of electrons emitted from the plate was 2.26 eV.

Calculate the maximum wavelength of radiation that can cause the ejection of an electron from the surface of this plate.

6 Problem – Brick and Styrofoam

The house has a brick wall with a thickness of $L_1 = 25$ cm, and a large area. The building was insulated from the outside with a layer of Styrofoam with a thickness of $L_2 = 30$ cm. Styrofoam sticks to the brick. A constant temperature is maintained inside the house $T_1 = 25$ °C. The outdoor air temperature is $T_2 = 12$ °C. Assume that the thermal conductivity of the brick is $k_1 = 0.9$ W/(K·m), and of Styrofoam $k_2 = 0.04$ W/(K·m).

Calculate the temperature T_b at the brick-Styrofoam border in a steady state, away from the edges of the wall.



7 Problem – Impact of irradiation

A new procedure for irradiating cancer cells was investigated. The irradiation should support a drug therapy. Without irradiation 130 out of 220 cell colonies were destroyed. However, in the experiment in which the new irradiation procedure was applied, out of 12 tested colonies as many as 11 colonies were destroyed, and only one colony survived. Assume that very similar, isolated from each other, cell colonies were used in the study.

Calculate the probability of obtaining such a result as in the experiment with the new procedure, assuming that the irradiation has no effect on colony survival probability.

Problems 8–13 are more difficult.

Send solutions of only two of them!

For each of these solutions you can get 6 points.

8 Problem – Toss

The ball with a mass of $m = 0.25$ kg was tossed vertically upwards. At the start, $t_0 = 0$ s, the ball moved with a speed of $v_0 = 8$ m/s. The ball is affected only by: gravity and air resistance; in particular, ignore buoyancy. Gravitational acceleration at the event site $g = 9.8$ m/s². Assume that the value of the air resistance force is equal to βv^2 , where v is the value of the ball speed, and $\beta = 0.04$ kg/m.

Calculate the time t_m , after which the ball will reach the highest point.

9 Problem – Proton in a magnetic field

Consider a proton moving in a vacuum, in an uniform magnetic field.

Case 1. Assume that the kinetic energy of the proton is constant and equal to $E_k = 10$ keV, the angle between the proton velocity vector and the magnetic induction vector equals $\alpha = 40^\circ$, and the magnitude of magnetic induction equals $B = 3$ T.

a) Calculate the pitch of the helical path along which the proton would travel in such an idealized scenario.

Case 2. In the experiment in which the proton velocity vector is perpendicular to the vector of magnetic induction, the kinetic energy of the proton decreased.

b) What is the reason of the observed change in the kinetic energy of the particle?

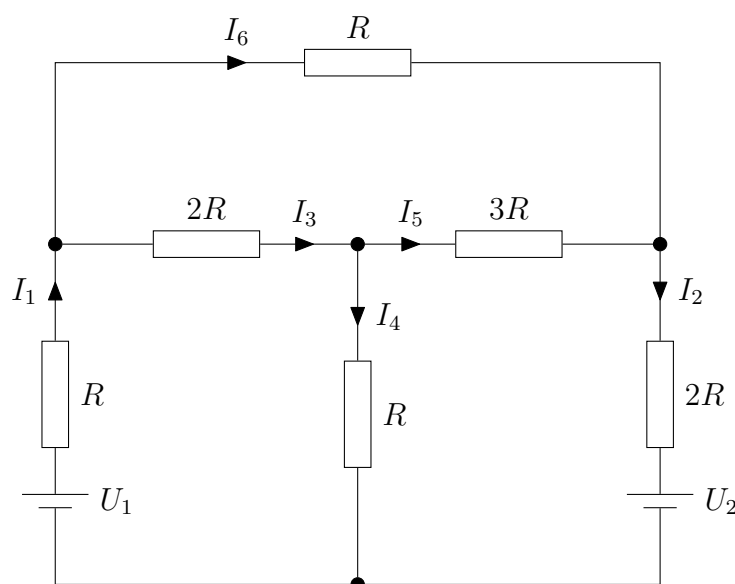
c) Describe, in a qualitative fashion, the path of the proton in this case.

10 Problem – Three loops

In the circuit shown in the following scheme resistances of the resistors are multiples of the resistance R . The voltages at the sources terminals are constant and equal to U_1 and U_2 , as shown. Currents are denoted by I_1, \dots, I_6 . In your solution use the same symbols as those used in the scheme.

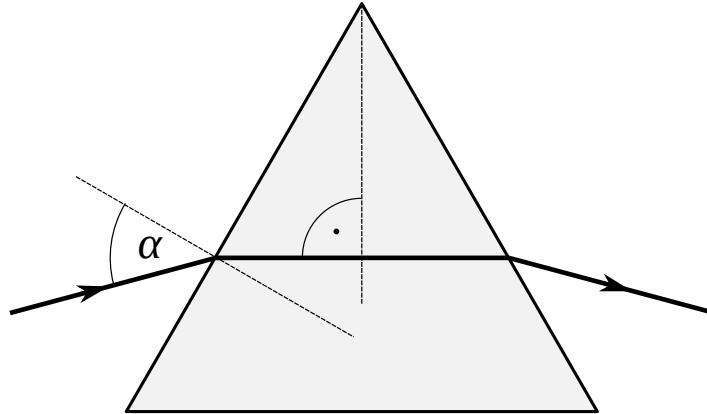
a) Find the dependence of I_3 on U_1 , U_2 , and R .

b) Calculate the value of I_3 for $U_1 = 5$ V, $U_2 = 3$ V and $R = 1$ k Ω .



11 Problem – Prism

A collimated beam of monochrome light falls on a prism located in a vacuum. The beam power is equal to $P = 1.5 \text{ W}$. The angle of incidence of the beam on the prism surface, measured with respect to the surface normal, is equal to $\alpha = 40^\circ$. The refractive index of the glass from which the prism is made is equal to $n = 1.6$. Inside the prism the beam is perpendicular to the bisector of the refracting angle of the prism. Prism surfaces have anti-reflective coatings, thus intensities of reflected beams are negligible. Assume that the power of the beam coming out of the prism is equal to the power of the incident beam.



Calculate the magnitude of the force that the light beam exerts on the prism in the above-described situation.

12 Problem – Electron in a cube

A quantum system was being excited and the light it emitted was studied. For the transition from the lowest, first excited state to the ground state, the measured wavelength of the emitted light, with uncertainty, was equal to $(687 \pm 2) \text{ nm}$.

A model was assumed in which the system can be described by means of an electron trapped in a stationary, empty cavity – a cube with the edge L . The electron cannot be outside this cube. Potential energy in the cavity was assumed to be 0. The probability density of finding an electron should go to 0 when approaching any of the walls. In the chosen Cartesian system, whose positive axes contain 3 edges of the cube, dependence of the wave function on the position along the X axis can be described by the $\sin(k_x x)$ factor, where k_x is a certain constant for a given state. Similarly for the Y and Z axes.

- Determine normalized wave functions – as functions of position and time – describing states with specific electron energy.
- Determine the formula specifying the possible energy values of the electron.
- Calculate L together with its uncertainty due to the uncertainty of the emitted-light wavelength measurement. Write down the value of L , taking into account the obtained uncertainty.

Hint. For positive integers p and r

$$\int_0^L \sin\left(p\frac{\pi}{L}x\right) \sin\left(r\frac{\pi}{L}x\right) dx = \frac{L}{2} \delta_{pr}$$

13 Problem – Cycle and adiabatic process

Part 1. One mole of a monatomic perfect gas undergoes the transformation shown in the pressure(p)-volume(V) diagram. The transformation from state B to state C is adiabatic. The pressure at point B is equal to $p_B = 64.1$ kPa. Other important values at points A, B, C can be read from the graph.

Calculate the efficiency of the heat engine in which the gas undergoes the described transformations.

Part 2. Starting from the first law of thermodynamics and the equation of state, derive the dependence of p on V in the adiabatic transformation of a monatomic perfect gas, which initially has volume V_0 and pressure p_0 .

