The scope of the exam
in enrollment to the Doctoral School of Exact and Natural Sciences
in the discipline of Mathematics

Structure of the exam: 8 problems in the discipline of Mathematics within the pool of 16 problems proposed together with the discipline of Computer Science. For the final result, 4 best evaluated problems are chosen from the 16 proposed problems.

The following list of topics is indicative only; exam problems may partly relate to other topics within the same general thematic range.

1. Mathematical analysis – functions of one variable

   Examples of topics: real and complex numbers and their properties, sequences and their limits, Bolzano-Weierstrass's theorem, Cauchy's condition, criteria of existence of a limit, series of real and complex numbers, convergence criteria for series, series conditionally and absolutely convergent, multiplication of series, continuity and uniform continuity of functions, properties of continuous functions defined on compact sets, Darboux's property, differential calculus of real functions of one variable, Rolle's i Lagrange's theorems, using derivatives and limit when graphing a function, series of functions, pointwise and uniform convergence, power series, radius and circle of convergence, Taylor's expansion, indefinite integral, Riemann integral, improper integral.

2. Mathematical analysis – functions of many variables

   Examples of topics: partial derivatives and directional derivative, gradient, Jacobian, extrema of functions of many variables, implicit functions, Lagrange's multipliers, theory of Lebesgue measure and integral, interchange of integration with the limit, Fubini's theorem, curvilinear and surface integrals, differential manifolds and differential forms.

3. Analytic functions

   Examples of topics: Analytic functions, Cauchy-Riemann's equations, Cauchy's integral formula, maximum principle, residua.

4. Probability theory and statistics

   Examples of topics: conditional probability, independence, random variables and their parameters, conditional expectation, Markov chains, types of convergence of sequences of random variables, laws of large numbers and the central limit theorem. Elements of statistics: estimators and their properties, testing hypotheses, linear regression.

5. Geometry and linear algebra

   Examples of topics: determinants and linear equations, linear and affine spaces, linear transformations, eigenvalues and eigenvectors, Jordan's theorem, bilinear and quadratic forms, inner products, Sylvester's criterion.

6. Algebra

   Examples of topics: groups, cyclic groups, groups of permutations, group homomorphisms, kernel, normal subgroup and quotient group, Lagrange's theorem about the order of a subgroup, commutative rings, ideals, maximal and prime ideals, homomorphisms of rings, zero divisors, invertible elements, field of fractions, fields, prime field, characteristic of a field, algebraically closed field, fundamental theorem of algebra, roots of unity.

7. Topology

   Examples of topics: metric and topological spaces, methods of defining a topology, Tikhonov's theorem, continuous mappings, Tietze's theorem, connected spaces, compact spaces, complete spaces, Cantor set and its properties, Baire's theorem, Banach's and Brouwer's fixed point theorems, fundamental group, compact surfaces.

8. Ordinary differential equations

   Examples of topics: existence and uniqueness of solutions of ordinary differential equations, solving ordinary differential equations of one real variable, linear ordinary differential equations and sets of linear differential equations with constant coefficients and solving them, sets of linear equations with variable coefficients and fundamental matrix, stability of solutions.

9. Functional analysis

   Examples of topics: Banach space, functionals and linear operators, dual space, Hilbert space, functional spaces L^p, spaces of continuous functions.