Doctoral School of Exact and Natural Sciences
Physical Sciences – written exam

Instruction manual. Solving the problems, present the reasoning behind the result. Give your final numerical results with the accuracy of two or three digits, after rounding up appropriately, e.g.. $1.23456 \cdot 10^{-19} \approx 1.23 \cdot 10^{-19}$ or $1.2 \cdot 10^{-19}$. In your calculations, you can use approximate values of physical constants shown in the table below.

speed of light in vacuum	$c \approx 3.00 \cdot 10^8\,\mathrm{m/s}$	elementary charge	$e \approx 1.60 \cdot 10^{-19}\,\mathrm{C}$
Coulomb constant	$k_e \approx 8.99 \cdot 10^9\,\mathrm{N\,m^2/C^2}$	gravitational constant	$G \approx 6.67 \cdot 10^{-11}\mathrm{N\,m^2/kg^2}$
reduced Planck constant	$\hbar \approx 1.05 \cdot 10^{-34}\,\mathrm{J\,s}$	Planck constant	$h \approx 6.63 \cdot 10^{-34}\,\mathrm{J\,s}$
Avogadro number	$N_A \approx 6.02 \cdot 10^{23}\,\mathrm{mol^{-1}}$	proton mass	$m_p \approx 1.673 \cdot 10^{-27}\mathrm{kg} \approx 938.3\,\mathrm{MeV}/c^2$
Boltzmann constant	$k_B \approx 1,38 \cdot 10^{-23}\,\mathrm{J/K}$	neutron mass	$m_n \approx 1.675 \cdot 10^{-27}\mathrm{kg} \approx 939.6\,\mathrm{MeV}/c^2$
Rydberg unit of energy	$Ry \approx 13.6\,\mathrm{eV}$	electron mass	$m_e \approx 9.11 \cdot 10^{-31}\mathrm{kg} \approx 511\,\mathrm{keV}/c^2$

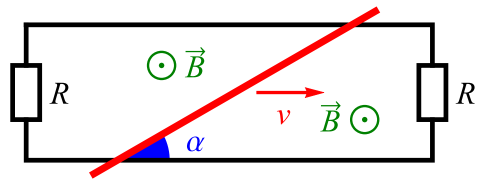
Problems 1-8 are the easier problems. Turn in solutions of only **four** of these problems. For each solution, you can be assigned up to 6 points.
Problems 9-14 are the harder problems. Turn in solutions of only **two** of these problems. For each solution, you can be assigned up to 8 points.

Prepare solutions for different problems on separate sheets.

EASIER PROBLEMS

Problem 1

Two parallel wires at a distance d are connected at both ends with resistors of resistance R. A magnetic field $\vec{B}$ is perpendicular to the plane of the wires; the field is constant and uniform. The wires are additionally connected with a bar which forms an angle $\alpha = 30°$ with the wires and moves with velocity v along the wires, as shown in the adjacent diagram. Find the current flowing through the bar.



Problem 2

The spectral lines of the deuterium atom are slightly shifted relative to the analogous lines of the hydrogen atom. What velocity will an astronomer, mistakenly convinced that it is the spectrum of a hydrogen atom moving radially relative to the observer, attribute to a stationary deuterium atom based on spectral studies? The mass of the deuteron (nucleus of deuterium) is $m_d = 1876\,\mathrm{MeV/c^2}$.

Problem 3

A particle of mass m moving in one dimension has, at a given time, the wavefunction $\psi(x) = Ae^{-\lambda|x|}$, where A and λ are positive constants.

a) Determine A.
b) Find the probability of finding that particle in the interval $-a < x < a$, where a is a constant.

Problem 4

A droplet of incompressible liquid has been excited to undergo small oscillations, which involve changes in the shape of the droplet relative to its spherical shape. The frequency f of these oscillations depends only on the surface tension of the liquid γ, its density ρ and the radius of the droplet r. Provide the form of this dependence up to a multiplicative constant.

Problem 5

In most nuclear medicine procedures that use gamma radiation, the radioactive isotope of technetium, $^{99m}\mathrm{Tc}$, is used, which has a half-life of $T_{1/2} = 6$ hours. After what time will only 1/18 of the initial sample of this isotope remain?

Problem 6

A patient has been administered a drug whose diffusion coefficient at body temperature ($T = 37°\mathrm{C}$) is $D = 10^{-9}\,\mathrm{m^2/s}$.

a) How much time is needed on average for a drug molecule to travel through the cell membrane and reach the center of a spherical cell with a radius $r = 1\,\mu\mathrm{m}$?
b) Calculate the radius of the drug molecule, assuming it is spherical and given that the viscosity of the cytoplasm at $T = 37°\mathrm{C}$ is $\eta = 1.5\,\mathrm{mPa \cdot s}$.

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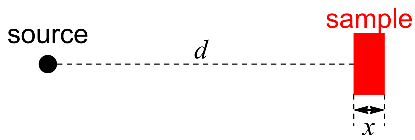
Problem 7
For the purpose of conducting a survival test, 200 cells were used, which were seeded onto two Petri dishes. One dish contained 100 control cells that were not exposed to any radiation. The other dish contained 100 cells irradiated with a dose $D_0 = 2\,\mathrm{Gy}$. After two weeks of incubation, 75 colonies formed on the control dish, and 20 colonies formed on the dish with the irradiated cells. Calculate the radiation dose D for which the cell survival fraction is equal to SF=37%, assuming that the survival curve is described by an exponential function of the dose. Each colony originates from a single cell seeded on the dish. The survival fraction (SF) is the ratio of the number of cells that survived irradiation to the number of non-irradiated cells that would have survived exposure to radiation.

Problem 8
A point source of ^{57}Co with an activity of $A_0 = 100\,\mathrm{MBq} = 10^8\,\mathrm{s}^{-1}$ emits gamma radiation quanta with an energy of $E_\gamma = 125\,\mathrm{keV}$. This radiation is incident on a sample with a thickness of $x = 20\,\mu\mathrm{m}$, surface area $S = 1\,\mathrm{cm}^2$ and a density equal to the density of water $\rho_w = 1000\,\mathrm{kg/m}^3$, located at a distance $d = 100\,\mathrm{mm}$ from the source. Between the source and the sample, there is air with negligible density and negligible gamma radiation absorption coefficient.



The attenuation coefficient of γ radiation with energy $E_\gamma = 125\,\mathrm{keV}$ in water is $\mu_{\mathrm{en}} = 2.65 \cdot 10^{-2}\,\mathrm{cm}^2/\mathrm{g}$. Calculate the dose rate for this sample, knowing that the absorbed dose is the average energy lost by the radiation and absorbed by the medium through which the radiation passes, per unit mass of that medium. The dose rate is related to the dose and time in a way well known to physicists.

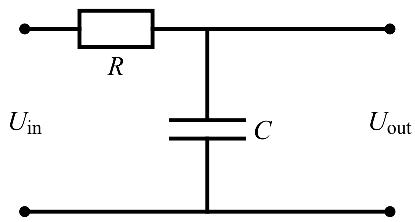
HARDER PROBLEMS

Problem 9
From the edge of a tower, at a height H above the ground, Galileo tries to drop lead balls, each with a mass m, as precisely as possible to hit a designated target. He has the most precise equipment available. Based on the Heisenberg uncertainty principle, estimate the typical distance from the target where Galileo's balls would land, assuming no external phenomena or physical processes can disturb their flight.

Problem 10
A biker rides her motorcycle, moving on a circular path of radius R. She is applying maximum acceleration, starting from the rest, to achieve the maximum velocity, so she is constantly on the verge of skidding. How much distance does she have to travel from the start before she reaches the maximal velocity? The friction coefficient between tires and asphalt is μ, the mass of the biker is m and the gravitational acceleration is g.

Problem 11
The RC circuit shown in the adjacent diagram consists of a resistor with resistance $R = 1\,\mathrm{k}\Omega$ and a capacitor with capacitance $C = 0.4\,\mu\mathrm{F}$.

a) For a sinusoidally varying input voltage with frequency f and amplitude U_{in} applied to the input of the circuit, determine the amplitude of the output voltage U_{out} as a function of R, C and f.
b) For the given values R and C determine the cutoff frequency f_{g}, for which the transfer function $T = |U_{\mathrm{out}}/U_{\mathrm{in}}|$ is equal to $1/\sqrt{2}$.



Problem 12
Three identical non-interacting particles of spin 1/2 are inside a cube with side length L and move freely within this region. Determine the ground state energy of this system.

Problem 13
Two distinguishable particles of spin 1/2 interact via the folowing Hamiltonian:

$$\hat{H} = \lambda\,\hat{\vec{S}}_1 \cdot \hat{\vec{S}}_2\,,$$

where $\hat{\vec{S}}_1$ and $\hat{\vec{S}}_2$ are the spin operators of the first and the second particle, respectively, and λ is a constant. Find the energy levels of the system and the corresponding eigentstates.

Problem 14
A spherical particle with a radius $R = 20\,\mu\mathrm{m}$ and density $\rho = 1750\,\mathrm{kg/m}^3$, initially resting on the surface of a vessel filled with water at a depth $h = 20\,\mathrm{cm}$ and temperature $T = 20°\mathrm{C}$, falls under the influence of a gravitational field with an acceleration $g = 9.81\,\mathrm{m/s}^2$. After what time will the particle reach the bottom of the vessel? The density and viscosity of water at $T = 20°\mathrm{C}$ are $\rho_w = 1000\,\mathrm{kg/m}^3$ and $\eta = 0.89\,\mathrm{mPa \cdot s}$, respectively. The drag force acting on a sphere of radius r moving with velocity v in a liquid with viscosity η is given by $F_{\mathrm{drag}} = 6\pi\eta r v$.